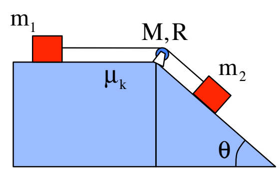
Problem 10.40

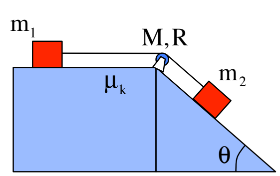
The block system is frictional and the massive pulley is assumed to be frictionless and can be approximated as a disk.



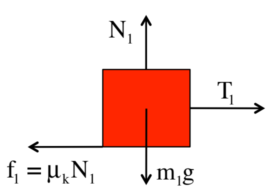
NOTE: This problem is designed to identity two bits of information for you. First, that you can write TWO Newton's Second Law equations, one for *translational motion* and one for *rotational motion*, for a system like this. The emphasis is on TWO, which is to say: In a N.S.L. situation, you CAN'T *sum torques* and *sum forces* *in the same relationship* and expect it to make any sense (they don't even have the same units!). Second, when dealing with a massive pulley, not only does the pulley have rotational inertia (in this case, that of a disk) but also, because the net torque on the pulley has to be non-zero (something that wouldn't be the case if the tensions on either side were the same), the tensions on either side of the pulley ARE NOT THE SAME and, as a consequence, must be treated like two individual unknowns. (Kind of a run-on sentence, but you get the idea.)

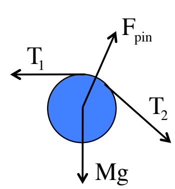
With those two insights, let us proceed.

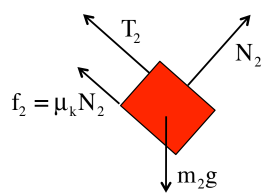
a.) Draw a f.b.d. for each of the elements of the system.



I've arraying the bodies and their associated forces so you can easily see the common forces (the action/reaction couples). Each such pair ALWAYS uses the same symbol for their force magnitude. Also, note that in many instances, *where* a force acts now matters (due to the way torque calculations are made), and that because the pulley is stationary, the sum of the forces on it must be zero (hence the need to include a pin force).

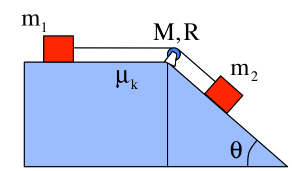






NOTE: THIS IS WHERE YOU STOP if asked to draw a f.b.d. on an AP test. That is, DON'T break any forces into their component parts.

b.) Derive an expression for the *acceleration* of the system.



Taking each body (pulley included) in turn and adding in coordinate axes, we get:

for m_1

$\sum F_v :$

$$N_1 - m_1 g = m_1 a_y \quad (a_y = 0)$$

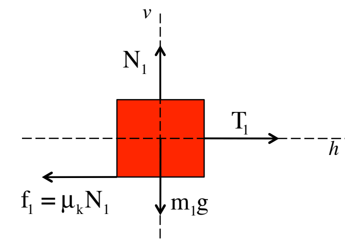
$$\Rightarrow \quad N_1 = m_1 g$$

$\sum F_h :$

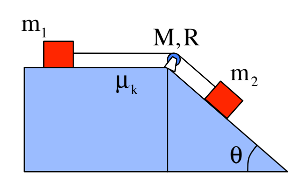
$$-\mu_k N_1 + T_1 = m_1 a$$

$$\Rightarrow \quad -\mu_k (m_1 g) + T_1 = m_1 a$$

$$\Rightarrow \quad T_1 = m_1 a + \mu_k (m_1 g)$$



Minor Note: If m_1 accelerates to the right, then m_2 must accelerate *down* the incline in what I've defined (via the coordinate axis I've set up) as the *negative y-direction*. Because the acceleration terms "a" is supposed to be a magnitude, I will have to unembed "a's" negative sign when writing out N.S.L. in the *y-direction*. As such:



for m_2

$\sum F_y :$

$$N_2 - m_2 g \cos\theta = m_2 a_y \quad (a_y = 0)$$

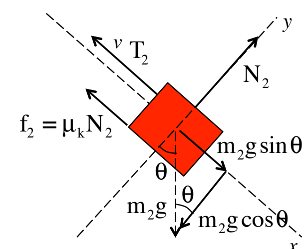
$$\Rightarrow \quad N_1 = m_2 g \cos\theta$$

$\sum F_x :$

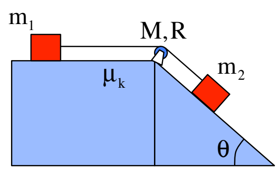
$$-\mu_k N_2 - T_2 + m_2 g \sin\theta = m_2 a$$

$$\Rightarrow \quad -\mu_k (m_2 g \cos\theta) - T_2 + m_2 g \sin\theta = m_2 a$$

$$\Rightarrow \quad T_2 = -m_2 a + m_2 g \sin\theta - \mu_k (m_2 g \cos\theta)$$



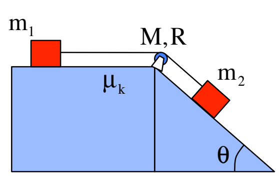
At this point, we have three unknowns, "a," "T_1" and "T_2." We need another equation which we can get by using the rotational version of N.S.L. on the pulley. There are a few things to note about this first.



1.) We don't need a Cartesian coordinate system attached to the pulley because we are not summing forces, we are summing torques.

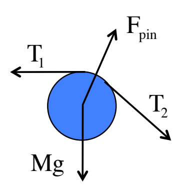
2.) The pulley's *angular acceleration* is *clockwise*, which means its *angular acceleration* vector is *negative*. Assuming we use the same protocol for rotating systems that we used for translating systems, we want all algebraic parameters to be magnitudes. That means we will have to unembed the negative sign from inside the α term on the right side of the rotational N.S.L. expression.

3.) We are modeling the pulley as a disk. A disk's radius vector is always perpendicular to a tangent to its edge. As the tension forces are tangent to the edge, the torque do to ALL tensions forces, no matter where they act on the pulley, will either be +TR or –TR, depending upon whether the torque motivates the body to angularly accelerate *counterclockwise* or *clockwise* (respectively).



4.) Because the string isn't slipping over the edge of the pulley, the acceleration of a point on the edge and the acceleration of the string will be the same. That means relationship between the pulley's *angular acceleration* and the *string's acceleration* will be $a = R\alpha$.

5.) And finally, noting that the *moment of inertia* of a disk is $I_{disk} = \frac{1}{2}MR^2$, we can utilize the tension expressions previously derived and write:



AND OH, MY, WE ARE GOING TO NEED A WHOLE LOT MORE ROOM TO FIT ALL OF THIS IN:

$\sum \Gamma_{pin}:$

$$T_1R - T_2R = -I_{pin}\alpha$$

$$\Rightarrow [m_1a + \mu_k m_1 g]\cancel{R} - [-m_2a + m_2 g\sin\theta - \mu_k(m_2 g\cos\theta)]\cancel{R} = -\left(\frac{1}{2}M\cancel{R^2}\right)\left(\frac{a}{\cancel{R}}\right)$$

$$\Rightarrow m_1a + \mu_k m_1 g + m_2a - m_2 g\sin\theta + \mu_k(m_2 g\cos\theta) = -\frac{M}{2}a$$

$$\Rightarrow m_1a + m_2a + \frac{M}{2}a = -\mu_k m_1 g + m_2 g\sin\theta - \mu_k(m_2 g\cos\theta)$$

$$\Rightarrow \left(m_1 + m_2 + \frac{M}{2}\right)a = [-\mu_k m_1 g + m_2 g(\sin\theta - \mu_k\cos\theta)]$$

$$\Rightarrow a = \frac{[-\mu_k m_1 g + m_2 g(\sin\theta - \mu_k\cos\theta)]}{\left(m_1 + m_2 + \frac{M}{2}\right)}$$

$$\Rightarrow a = \frac{[-(.360)(2.00\text{ kg})(9.80\text{ m/s}^2) + (6.00\text{ kg})(9.80\text{ m/s}^2)(\sin 30^\circ - (.360)\cos 30^\circ)]}{\left((2.00\text{ kg}) + (6.00\text{ kg}) + \frac{(10.0\text{ kg})}{2}\right)}$$

$$= .309\text{ m/s}^2$$

b.) Determine the tensions.

$$T_1 = m_1a + \mu_k(m_1g)$$
$$= (2.00\text{ kg})(.309\text{ m/s}^2) + (.360)(2.00\text{ kg})(9.80\text{ m/s})$$
$$= 7.67\text{ N}$$

$$T_2 = -m_2a + m_2 g\sin\theta - \mu_k(m_2 g\cos\theta)$$
$$= -(6.00\text{ kg})(.309\text{ m/s}^2) + (6.00\text{ kg})(9.80\text{ m/s})\sin 30^\circ - (.360)(6.00\text{ kg})(9.80\text{ m/s})\cos 30^\circ$$
$$= 9.22\text{ N}$$

And to check:

$$T_1\cancel{R} - T_2\cancel{R} \overset{?}{=} -\left(\frac{1}{2}M\cancel{R^2}\right)\left(\frac{a}{\cancel{R}}\right)$$

$$\Rightarrow (7.67\text{ N}) - (9.22\text{ N}) = -\frac{1}{2}(10.0\text{ kg})(.309\text{ m/s}^2)$$

$$\Rightarrow -1.55\text{ N} = -1.55\text{ N}$$

IT WORKS!